UNIVERSITÀ DEGLI STUDI FIRENZE
DINFO DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

Inversion Methods for Tropospheric Water Vapor Retrieval by means of a Constellation of LEO Satellites: the SATCROSS Contribution

Argenti, Fabrizio (1); Mazzinghi, Agnese (2); Facheris, Luca (1); Cuccoli, Fabrizio (2); Antonini, Andrea (3); Rovai, Luca (3)

(1) Department of Information Engineering (DINFO), University of Florence - Via di Santa Marta 3 - 50139 Florence - ITALY
(2) CNIT- RASS, National Inter-University Consortium for Telecommunications, Italy c/o DINFO
(3) Consorzio LAMMA c/o CNR IBE - Via Madonna del Piano, 10, 50019 Sesto Fiorentino – ITALY

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ABSTRACT

The Normalized Differential Spectral Absorption (NDSA) method has been proposed to retrieve the Water Vapor (WV) content in the troposphere. Since NDSA produces Integral WV (IWV) measurements, inversion methods are needed to achieve the spatial distribution of the WV.

In this study, we investigate on inversion methods that can be applied to the acquisition system studied in the SATCROSS project. The system consists of a constellation of Low Earth Orbit (LEO) satellites, orbiting in the same plane and along the same direction, in which part of the satellites carries onboard transmitters (Tx) operating in the K and Ku frequency bands and the remaining are equipped with receivers (Rx). By applying the NDSA approach along the Tx-Rx links that can be deployed, each of them crossing the troposphere, a set of IWV measurements can be achieved, whose inversion yields the WV content in the annular region contained in the orbital plane of the satellites.

The particular geometry of the inversion problem at hand perfectly fits the so-called *external reconstruction problem*, proposed for the inversion of the Radon transform of a field when such transform is available only in the external part of a body. In this study, we investigate on its application to the IWV inversion problem and compare its performance with that of already proposed methods.

Introduction

Normalized Differential Spectral Absorption (NDSA)

The NDSA technique is a measurement approach that consists in transmitting – along a microwave propagation path – two sinusoidal signals with close frequencies f_1 and f_2 ($f_1 > f_2$), with $\Delta f = f_1 - f_2$, and computing the spectral sensitivity S as

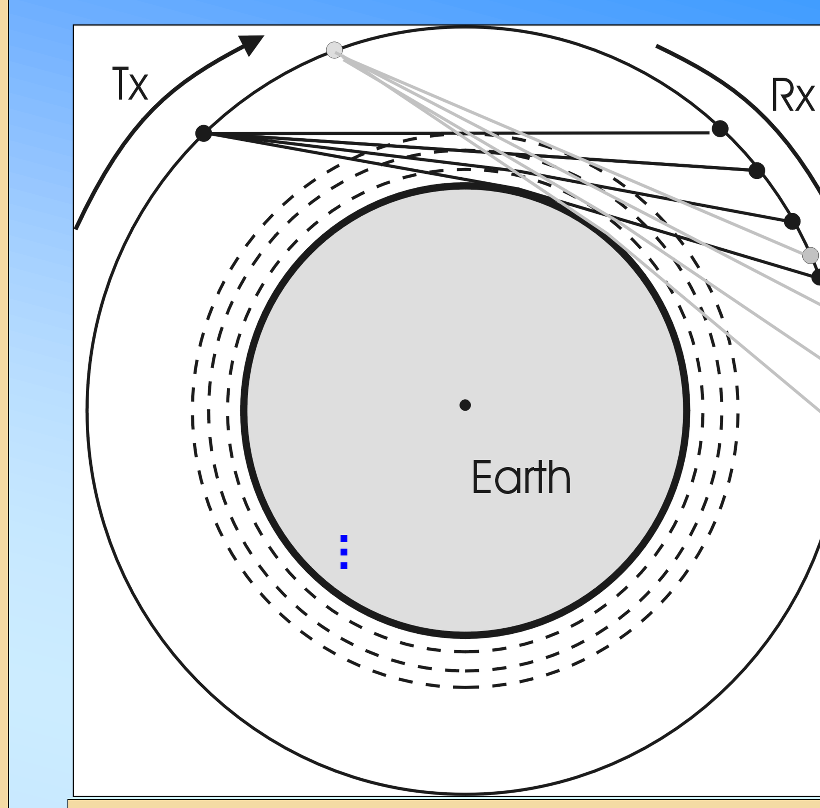
$$S = \frac{P_2 - P_1}{\Delta f \; P_2}$$

where P_1 and P_2 are the powers of the two received tones.
It can be shown that:

1) the parameter S is closely related to the total content of water vapor along the radio link (integral WV, IWV), when the center frequency $f_0 = (f_1 + f_2)/2$ belongs to the Ku-K bands;
2) the NDSA measurement is robust against degradations due to the scintillation phenomenon.

Measurement system based on a constellation of Low Earth Orbit (LEO) satellites: the SATCROSS project contribution

The SATCROSS project, funded by the Italian Space Agency (ASI), investigates on the application of the NDSA measurement approach to a particular acquisition system composed by a constellation of LEO satellites orbiting in the same plane and in the same direction. Part of the satellites carries onboard transmitters (Tx) operating in the K and Ku frequency bands and the remaining are equipped with the receivers (Rx). By applying the NDSA approach along the Tx-Rx links that can be deployed, each of them crossing the troposphere, a set of IWV measurements can be achieved, whose inversion yields the WV content in the annular region contained in the orbital plane of the satellites.



Co-rotating LEO satellites

The External Reconstruction Tomographic Algorithm (ERTA)

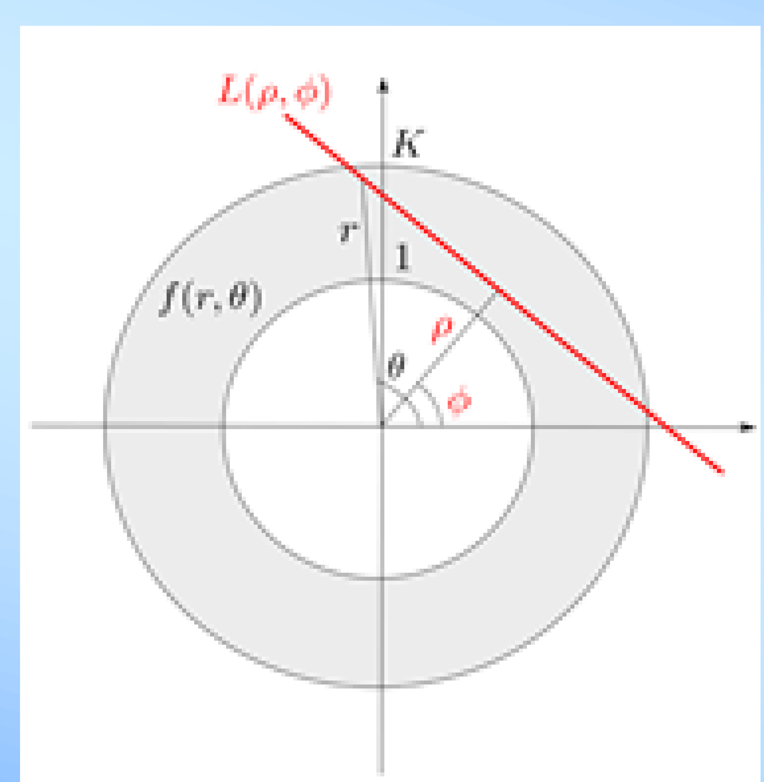
The Radon transform and the exterior reconstruction problem

- Let $f(r,\theta)$ denote the WV function to be reconstructed expressed in polar coordinates. The set of IWV measurements represents a sampling of the Radon transform computed on an annular region, i.e.,

$$g(\varrho,\phi) = \int_{L(\varrho,\phi)} f(r,\theta) ds$$

where L is a line identified by the couple (ϱ,ϕ)

- It can be shown [Cormack 1963 and 1964][Perry, 1977][Quinto, 1983 and 1988] that the values of the function $f(r,\theta)$, for $r > r_0$, can be retrieved from those of its Radon transform $g(\varrho,\phi)$ for $\varrho > r_0$



The geometry of ERTA (normalized with respect to Earth radius)

Basic ERTA results

- The functions $f(r,\theta)$ and $g(\varrho,\phi)$ are periodical in the angle variables θ and ϕ, respectively, so that they can be expanded as Fourier series, that is

$$f(r,\theta) = \sum_{n=-\infty}^{\infty} f_n(r) e^{jn\theta}, \qquad g(\varrho,\phi) = \sum_{n=-\infty}^{\infty} g_n(\varrho) e^{jn\phi}$$

- It can be shown that each term of the two series can be further decomposed by means of bases of functions, whose elements $f_{n,l}(r,\theta)$ and $g_{n,l}(\varrho,\phi)$, $l \in N$, are described by Jacobi polynomials and such that the Radon transform maps such functions onto each other, i.e., $f_{n,l}(r,\theta) \rightarrow g_{n,l}(\varrho,\phi)$, and viceversa
- The elements $f_{n,l}(r,\theta)$ and $g_{n,l}(\varrho,\phi)$ are orthogonal in the space domain and in the Radon domain, respectively, if particular weight functions are used to define the inner product in such spaces

Reconstruction Algorithm

1) Decompose the IWV data (Radon domain), converted from acquired NDSA measurements, by using the $g_{n,l}(\varrho,\phi)$ functions
2) Reconstruct the space domain function $f(r,\theta)$ by using the coefficients of the expansion and the mapping $g_{n,l}(\varrho,\phi) \rightarrow f_{n,l}(r,\theta)$

Simulation results

Acquisition system and satellite constellation parameters

- Spherical Earth with radius equal to 6378 km;
- 1 Tx satellite and 5 Rx satellites;
- Satellite altitude equal to 273 km;
- Constant angular speed with a revolution period of 90 min;
- Integration time at the receiver: Ts = 1 s.

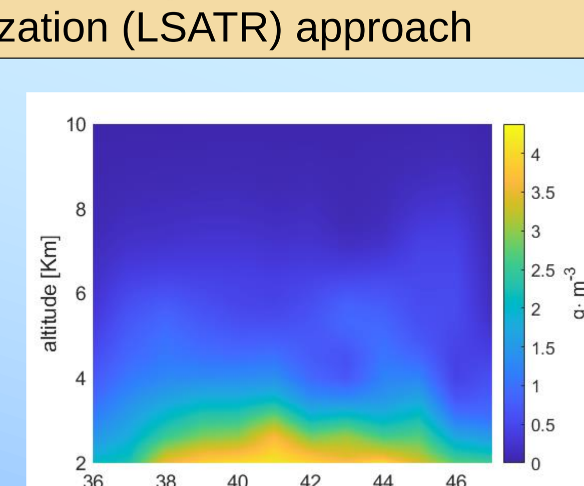
Radio link and noise parameters

- Transmitted power: 3 dBW on each NDSA tone;
- Frequency channels for NDSA measurement: 17, 19, and 21 GHz, with spectral separation Δf = 0.2 GHz;
- Transmit and receive antenna gains: 26.4 dB;
- Scintillation: log-amplitude standard deviation $\sigma_\chi = 0.3$; zero-lag correlation between the two NDSA frequencies $\rho = 0.85$; scintillation bandwidth $B_\chi = 0.1$ Hz;
- Thermal noise: equivalent noise temperature at the receiver = 25.3 dBK.

Reconstruction example

- We used 'true maps' generated through the WRF (Weather Research and Forecasting) numerical weather prediction model, initialized with the IFS (Integrated Forecasting System) model of the ECMWF, with a vertical sampling interval of 125 m and a horizontal sampling of 3 km
- We compared the ERTA method with the Least Squares Algorithm with Tikhonov Regularization (LSATR) approach

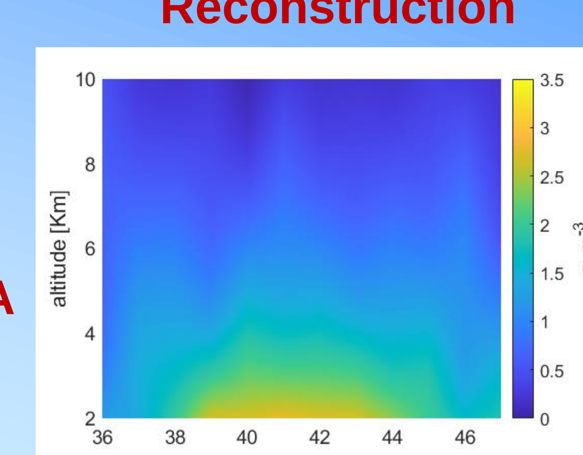
True map

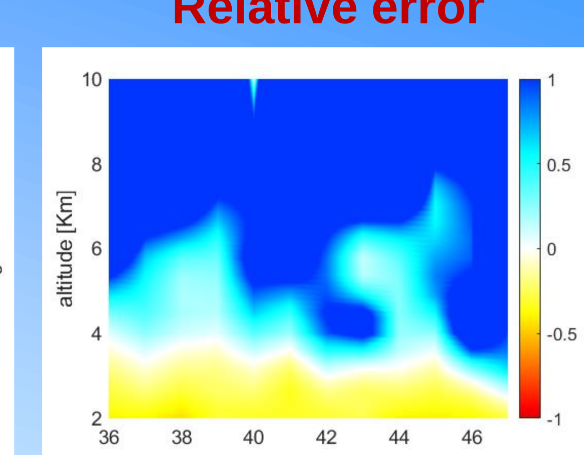


Analysis of 27 July 2020 03:00pm

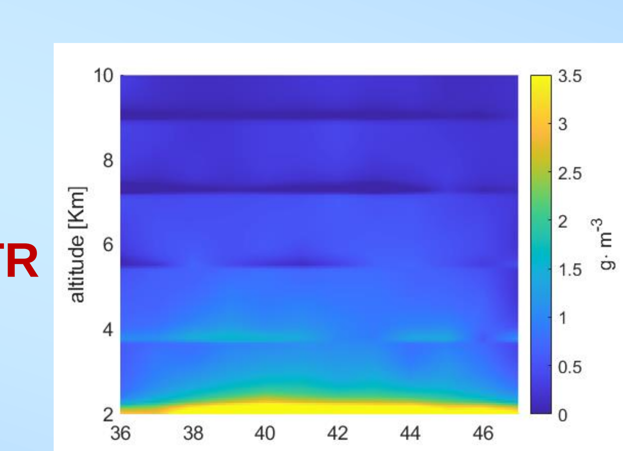
Reconstruction | **Relative error**

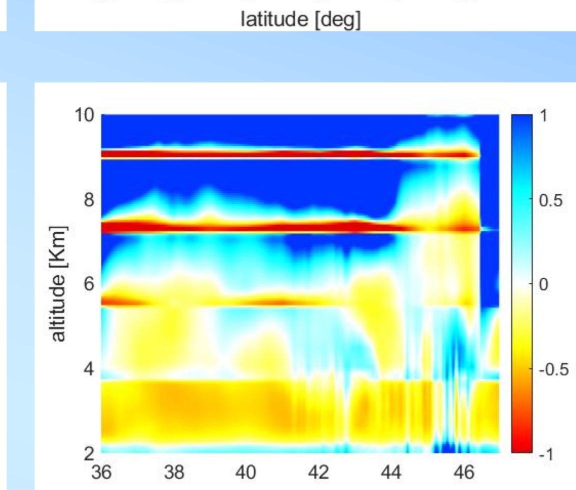
ERTA





LSATR





CONCLUSIONS

In this study, we investigated on the application of the exterior reconstruction problem to the case of IWV measurements derived from a constellation of LEO satellites and the NDSA approach. The ERTA approach has been tested by using a simulator that takes into account realistic impairments as well as synthetic atmospheric scenarios, which allow the reconstruction error to be evaluated. The results have been compared to those obtained with a least squares approach with Tikhonov regularization, proposed by the authors and available in the literature. The outcomes of the simulations highlight a pretty different distribution of the retrieval error and, from a quantitative point of view, a better performance of the ERTA approach in the low troposphere.

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